A Theory of Community Formation and Social Hierarchy

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Abstract

In many new venues for exchange, populations are large and the legal ordering is weak, making it easy to cheat and thenceforth to avoid trading partners. We analyze equilibria where cooperation can be sustained through endogenous formation of subgroups or "markets" where individuals concentrate their trades. To engender loyalty and deter individuals from a "cheat and leave" strategy, we consider alternative approaches including market-specific investments and the creation of social hierarchies. We highlight a type of social hierarchy wherein trust increases with seniority and agents probabilistically rise to higher levels of status. Thus the value of remaining loyal to a group increases over time and senior agents, because they have the least incentive to cheat, trade more.

An important robustness feature of these "hierarchical" equilibria is that trade with trustworthy agents in one group is not restricted solely in order to maintain cooperation in other groups; instead, at all points in the game agents are trusted up to the point where they would "cheat and run." Cooperation in hierarchical markets thus can survive even in the presence of alternative "institution-free" groups that offer full cooperation and free entry. In fact, the presence of hierarchical markets may undermine cooperation in groups with alternative institutions.

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1 Introduction

> In a deal, you give and take. You compromise. Then you grab the cash and catch the next train out of town ...
>
> - Irving "Swifty" Lazar.[1]

From pastoralists in northern Nigeria to software programmers in Silicon Valley, those that seek to gain from trust and trade are often forced to rely on their own devices. As transitional economies open to markets, people migrate to cities, and new technologies facilitate new ways to interact, low third party observability and a large set of potential trading partners allow opportunities to cheat and run that can undermine trust and cooperation.[2]

This paper studies cooperation based on reciprocity in large groups when the legal ordering is weak and opportunistic behavior is difficult to observe publicly, preventing third party sanctions. These broad features appear to characterize a wide range of settings, from new marketplaces in transitional economies to online open source communities. In such settings, informal, self-enforcing institutions often emerge that help sustain trust.[3]

Formally, we model communities or markets as being potential loci for interaction. Each period, individuals select a market, observe who else has chosen to attend that market and engage in "trust games" with partners within that group. The principal in the trust game chooses how much to trust the agent (the scale of the trade), and the agent chooses whether to work or shirk. In our baseline model, matching within a market is random, capturing the idea that selecting a market inherently commits players to interacting with all of its members. However, we also consider an alternative where players can choose specific partners for exchange and thus also to actively avoid others within a market. The random matching model may be more appropriate if each member of a community has some specialty, and others' needs for those specialties arise randomly; it may also apply if we think of the trust games as consisting of small favors or courtesies, where opportunities for trust arise due to proximity or in the course of every day transactions. The choice-based model applies better to situations where

[1] Cited in McMillan [37],pg.53.

[2] For example, in a representative survey of commercial contracts in the major cities of Ghana, Marcel Fafchamps [13] finds that the ability to cheat and start over in a new market also plays a prominent role in the failure of contracting. He provides the following illustration:

> A fish trader purchased a quantity of fish on credit with the intent of selling it in villages. Later, she discovered that the fish was not selling well because farmers were having a bad year. She therefore decided to leave the fish business altogether and to reorient her trading activities to textiles. To start the textile operation she needed funds. She was currently unknown to textile suppliers and could only purchase goods on a cash-and-carry basis. All her working capital was tied up in fish, which she did not sell well. So the only way for her to get started in textiles was not to pay for the fish she had bought on credit ... Since she was not intending to re-enter the fish business, she did not mind angering the fishmongers [13][pg.445].

[3] For example, the opening up of markets in transitional economies– particularly Russia, China, Poland and Vietnam– led to a surprising wave of small-scale entrepreneurship in the 1990s. In each of these environments, third party enforcement was extremely limited, and entrepreneurs were forced to rely on "self-help"- informal institutions- to substitute for formal systems of contract enforcement (McMillan and Woodruff [38, 39]). See also Banerjee and Duflo [4]'s survey of contracting in the Indian software industry and the examples below.

the members of the market are fairly homogeneous in their ability to complete the types of transactions where trust is required, and the transactions are not too time-sensitive so that the desired partners can be located.

We are particularly interested in the role of "endogenous institutions," by which we mean strategies where individuals differentiate among trading partners based on factors other than their own trading history, such as history of attendance in a market. If individuals become "loyal" to markets, repeated interaction between group members can provide incentives for cooperation within the group. The challenge then is to understand the incentives that maintain group loyalty, despite the ability to cheat and to start afresh in a new group. Indeed, loyalty by others makes it easier to avoid particular individuals by selecting other markets.

We discuss a number of plausible institutions that can induce loyalty. The creation of entry barriers based upon observable, upfront investments along "cultural" or market-specific dimensions are one alternative; we propose other alternatives where a period of *time* without trade substitutes for market-specific investment. These alternatives are feasible whenever agents have access to additional information about other members of a market, such as their record of attendance in the group or their "seniority" level. We show that social hierarchies based on seniority engender loyalty, thus providing the motivation for recording seniority levels or attendance in the first place.

We consider a series of questions in this context. First, which plausible institutions "do well" in our motivating environment where players have the option to cheat and start afresh in another market, perhaps at no cost? Second, how do institutions based on observable market-specific "cultural" investments compare in terms of efficiency to those based on social hierarchies? Third, can a social structure with more than two individuals interacting sustain more trust and exchange than that accomplished with pairwise relationships, absent any technological reason for group production? Fourth, do these equilibria appear artificial in the sense that they distort exchange by trustworthy partners in one group solely in order to sustain cooperation in other groups? Fifth, which social conventions can sustain within-group cooperation in the face of institutional changes in other groups, such as the lowering or elimination of entry barriers?

We begin our formal analysis by briefly describing two benchmarks. In a *peddler's equilibrium*, individuals randomize over markets. With large populations, this requires high degrees of patience to sustain cooperation. Alternatively, individuals might try to coordinate exchange within smaller markets of "loyal" agents. However, since the possibility to cheat and to costlessly leave a market still exists, such loyalty typically cannot be sustained without additional "institutions".

We show that specific investments in a shared group identity can solve the "cheat and run" problem by lowering the individuals' outside options. Such investments can take a number of forms, depending on whether there are exogenous dimensions through which groups may assert distinction between one another and conformity within themselves (e.g. language, culture, physical location, religious or ethical codes (Becker and Stigler [6])). For this class of equilibria, the group size for which it is easiest to sustain cooperation is two: partnerships maximize the sanction from cheating. Thus, absent exogenous reasons for group production, there are no gains in having bigger groups.

Next, we show that time spent in a "purgatorial" state without trade or at lower levels of social hierarchies can also act as a specific investment and can thus be rationalized as a means to enforce cooperation. In a *purgatorial equilibrium* individuals reduce the scale (or do not trade altogether) with new members of a group for a pre-specified number of periods. The prospect of facing purgatory once more in a new group deters mobility and makes individuals loyal.[4]

Although equilibria based on specific investments as well as those based on purgatory seem to be consistent with a number of real-world examples, they also share two undesirable robustness properties. First, a group as a whole would be better off if it abandoned these institutions, because these requirements serve only to induce loyalty in other groups. Second, in these equilibria, newly arrived agents or individuals who have yet to make group-specific investments can actually be trustworthy (they would be willing to work if trusted), yet trust will be withheld.[5] Motivated by these concerns, we construct an alternative class of equilibria. A *hierarchical equilibrium* is closely related to a purgatorial equilibrium in that equilibrium strategies call for reduced trade with more junior members. However, instead of using a fixed length of time where an individual is at the "lower level," there may be multiple levels of predetermined size. Advancement from one level to the next occurs only if some individuals at the higher level leave or die. Empty places further up in the hierarchy are randomly assigned to individuals at lower levels with priority given to more senior members. This feature permits us to construct equilibria where trust increases at each level, but individuals at each level of the hierarchy are either fully trusted or trusted to the point where their incentive constraints bind: they would actually cheat and run if trusted further. We show how changes in the hierarchical structure (i.e. the number of levels and individuals at each level) affect both the existence and efficiency of equilibria. We find that vertical mobility should be neither "too low" nor "too high" in order for a hierarchical structure to sustain cooperation.

Finally, we consider a natural alternative to the way individuals are matched in a market, allowing them to choose their partners, rather than be randomly matched. This allows individuals to focus their trades on "more trustworthy" agents. We show that cooperation can emerge with agents discriminating on seniority, much as in the hierarchical setting with random matching. As in the random matching setting, it is possible to construct equilibria in which agents trade to the point where they are just indifferent between cooperating and cheating. Since matching is not random, inefficiencies due to "mismatches," that is, principals withholding trust to more junior agents, do not occur in equilibrium. The equilibrium is always fully efficient.

[4]Interestingly, larger groups reduce the externality that new members impose on others that might be matched with them. With only two individuals per market, if a partner dies, the survivor must undergo the whole length of purgatory once more with a new partner. Thus, the optimal group size will be bigger when social conventions require a period of purgatory to engender group loyalty.

[5]For example, in the last period of purgatory, an individual's continuation value is the same as that of senior individuals, yet the purgatorial equilibrium's strategies still call for reduced trade. This feature is also a source of concern in models of "status building" in evolutionary biology, in which costly donations at earlier stages help qualify individuals to be recipients later on (e.g. Nowak and Sigmund [40]).

Furthermore, such hierarchical groups, by providing seniors with relatively more trades, also provide incentives for cooperation that are robust to institutional changes in other groups. Social hierarchies can sustain cooperation in environments where groups based upon cultural entry barriers will fail. In the extreme, such environments include the presence of an institution-free market that immediately maintains full trust among its members. Hierarchical markets can in fact can lead to a breakdown in cooperation in such a market. Further, unlike social networks which are often person-specific, social hierarchies are "impersonal" and the patterns of cooperation in a social hierarchy can survive beyond the individuals that populate them. Thus, social hierarchies may generate robust and spontaneous order that allows cooperation in new venues for trade.

The question of how certain institutions sustain cooperation in large populations with weak third party enforcement naturally spans a wide number of important settings. The following illustrations, one drawn from open source software communities, the other from pastoral communities in Nigeria, show how our analysis can speak to a number of specific puzzles that might seem at first to be unrelated.

Online Communities Open Source Software (OSS) communities and internet discussion groups are increasingly important venues for exchange (see, Raymond [44], Lerner and Tirole [33], or Shah [45]). As of 2006 there were over one hundred thousand open source projects with over a million registered users. A central feature of online communities at first seems surprising from the perspective of standard incentive theory: individuals regularly contribute to a public good despite the lack of monetary incentives, large group size and the relative anonymity of the interactions. A closer look at these communities reveals that it is not purely public-spiritedness that motivates users however. Many groups have formal or informal hierarchies. In OSS communities, there is often a group of senior members known as "committers" who have the authority to incorporate both their own code into the project, as well as that of others. In addition, there is often an informal hierarchy, where more senior members will help answer questions for other senior members, but not junior members. In other internet discussion groups, it is common to show the date a user joined beside their posts.[6] Other types of designations (such as top reviewers for Amazon.com, or Most Valuable Professionals on Microsoft Forums) also help distinguish the senior members of a market. Our analysis helps to understand these phenomena.

Consider how OSS projects fit into the model outlined in the introduction. Even though in principle individual contributions may be publicly observable in many online communities, in practice it requires a substantial investment to learn about the quality of their work. One might need to carefully read a programmer's code. Some aspects of support and collaboration in OSS projects take place in private correspondence. It may thus be difficult for outsiders to ascertain the quality of interactions others are having in the market, and one may only learn about an

[6] As Microsoft's Office Forum web page indicates, "the information about your activity in the Community (such as how many posts you have contributed...) gives others a sense of your trustworthiness and a way to gauge how valuable your comments might be."

individual's behavior by trying to adopt the code or otherwise having a close interaction with someone.

Our model applies to a setting where programmers select whether to become involved in a particular project. Programmers may write code for different purposes. Individuals associated with the project have needs arise which may be met using code that others have written, and they may choose whether or not to invest in reading and trying to use the code (as opposed to writing from scratch or finding other sources). They may then have support questions. The author of the code can then choose whether to support the user, answer questions, etc.[7] The "scale" of trust on the part of the user can be interpreted in several ways: for example, the user might make use of only the basic features of the code, or the user might invest rewriting portions himself. The user observes whether the ex ante quality was high, and also observes whether support was provided. The author gets higher utility the more the user relies on the author's original code (rather than rewriting it), because competing versions of the same code are thereby avoided. In such settings, the observed seniority-based hierarchies of trust and support can be understood as institutions that engender cooperation.

Pastoralists in Nigeria Across the historical and developmental spectrum from open source communities, but sharing a number of economic features, Murray Last's [32] historical study of the conversion to Islam among Hausa traders in northern Nigeria chronicles the Islamicization of the Hausa in northern Nigeria. While the Hausa were originally mainly animist, **i**n the twentieth century, Islam began to make steady inroads into the Hausa population and by the 1970s, particularly in the most densely populated areas, almost all Hausa had converted to Islam. A surprising feature of the process of conversion was that the remaining Non-Muslim Hausa clans, who used to live in relatively close proximity, both to each other and to the emergent Muslim population, suffered a breakdown in trade amongst themselves. Non-Muslim clans ceased to trade with one another, and became dispersed autarkic units. Trade among non-Muslims could only effectively be sustained in isolated areas away from the Muslim population (Greenberg [19]). The Hausa example illustrates a more general phenomenon: population increase and trade expansion often brings with it a breakdown of cooperation within traditional non-hierarchical lineages and the development of social hierarchies (sometimes as a precursor to the formation of centralized states) has been noted as a common feature **in** the development of stateless societies by ethnographers of Western Africa (Horton [24]).[8]

Our analysis also helps to explain these phenomena. First, Hausa Muslims appear to have

[7] Some of the author's choices may be made *ex ante*, such as deciding on the extent of documentation and code quality, modularity/adaptability, etc.

[8] See also the suggestive statistical evidence on centralisation and trade in pre-colonial African societies presented in Bates [5]. It has been argued that a key reason for conversion to Islam in other contexts was that Muslims enjoyed technological advantages, such as legal codes, or access to the Hajj, that facilitated trade (Ensminger [11], Jha [27]). While possibly true elsewhere, these advantages were arguably less clear among the 20th century Hausa than in other contexts. Furthermore, such technological improvements alone would not explain the breakdown of cooperation among non-Muslim Hausa–instead they might even be expected to improve such cooperation.

adopted a "social hierarchy" of a type mirroring that described in our model. As Last [32] describes, when they first convert to Islam, new Hausa Muslims spent one or two years where they still retain the ability to revert to their previous religion. During this "transitional" period, new Muslims tended to gain a small network of Muslim creditor-"friends" but were not trusted as much by Muslims at large. However, once the number of Muslim friends expanded sufficiently to reduce the attractiveness of exit, there tended to be a dramatic expansion of trust among other Muslims. Thus, becoming a "senior" Muslim conferred advantages over new Muslims in the number of creditors and the degree of trust (Last [32]).[9]

Next, as our analysis will show, the adoption of a social hierarchy by Muslims may have brought with it the promise of greater trade once a person became a "senior." We will show that such hierarchies can sustain cooperation even when alternative institutions can not, and that once an individual enters a hierarchical community, they may gain more from returning than they would from going to a community that trusts all participants equally. In contrast, had Muslims adopted social conventions that depended purely upon Islam-specific investments or "purgatory" to sustain cooperation, the presence of "institution-free" non-Muslim traders would have made it easy for Muslims to "cheat and run," potentially undermining cooperation among the emergent Muslim population.[10]

The rest of the paper proceeds as follows. In Section 2, we review related work. Section 3 introduces our formulation. Section 4 presents two benchmark equilibria. Section 5 and 5.4 introduce and compare alternative social conventions based upon identity investment and social hierarchies. Section 6 concludes.

2 Existing Theoretical Perspectives

The problem of sustaining trust has long been seen as a fundamental question in economic development, and economics more generally (McMillan [37]). While a simple model with increasing returns to scale may explain groups and hierarchical organizations may exist for a variety of reasons, including information processing, resource allocation, supervision and coordinating decision-making (e.g. Van Zandt [49], Maskin et al [36], Hart and Moore [22], Gibbons et al. [18]), the focus of our study is instead to understand the role that endogenous groups and

[9] As Last states:

> Hausa trade is essentially dependent on networks. Growth depends on the number of trading friends, not the cheapness necessarily of one's wares and the number of new customers one can thus attract. "Friends" are those to whom one gives goods on credit . . . The more creditors one has, the greater the turnover- and the greater the profit to one's creditors. Thus in a market there may be several traders selling identical articles, but the customer does not shop around comparing prices: he goes to his friend, who may or may not sell to him at a discount. . . [32][pg.240]

[10] It is possible that institution-free groups can also undermine third party enforcement. For example, Christopher Woodruff [52] finds evidence of information-sharing and third party enforcement among Mexican shoe manufacturers prior to the liberalization of trade. However, a significant aspect of his study is that this enforcement appears to break down as new opportunities to trade with the US emerge, allowing new "institution-free" trading partners outside the coalition.

social hierarchies can play in sustaining such trust.

Beyond the classic folk theorems, theorists traditionally focused on two types of mechanisms that overcome the trust problem: those that signal reputations and those that require third party enforcement. In reputation-based models, players learn about the others' type as honest or opportunistic as the relationship progresses. Equilibria in which more senior individuals are accorded more trust emerge as a result of these inferences (e.g. Sobel [46], Watson [50] and Ghosh and Ray [16]). Our formulation exhibits an analogous equilibrium dynamic without resorting to individuals' heterogeneity. We also differ from an important line of research on cooperation in groups that employs multi-lateral enforcement among delimited coalitions, following Greif [20] and Kandori [28], by examining settings where multi-lateral enforcement is not possible and populations can be arbitrarily large.[11]

Beginning with Klein and Leffler [29], the role of specific investments or "cultural capital" has assumed an important role in research into trust. For example, Iannaccone [25] applies the notion of specific investments to cults that provide club goods. There is an incentive to free-ride upon others' zealousness. In order to limit participation to the truly committed, religious practices, such as stigma and self-sacrifice develop to act as screening devices. Similarly, Fryer [15] allows for identity-specific investments among blacks and compares their effects on within-market trust with that of investing in general human capital or "acting white".[12] These works, however, take both the set of identities and prescriptions to be exogenous.

The intuition underlying the identity-investment equilibrium– using barriers to entry such as costly "gifts" into new relationships to sustain cooperation in old relationships– has been noted by a number of important studies (e.g. Kranton [30], Carmichael and MacLeod [9], Ramey and Watson [42]). The focus of these studies has however chiefly been two-agent partnerships.

Research has also begun to examine the role of such conventions for sustaining cooperation in groups. The role of membership fees in engendering loyalty to "insiders" with whom such costs have not been incurred has been explored by Board [8], and the role played by time in acting as such a membership fee– closely related to the purgatorial equilibrium– has been explored by Resnick [14] in the context of internet chat rooms. Sobel [47] analyzes a model where individuals in a large population form bilateral relationships. He contrasts relational contracts with formal contracting as mechanisms for sustaining trust. As in our model, it can be inefficient for partnerships to be exclusive in every period. Sobel's model has relationships that permanently "grow stale". In his model, relationships based on relational contracts may last

[11] Dixit's [10] study of trade expansion and enforcement is similar in motivation to ours. He uses a circular world as a geographical analogy for the costliness of information flows across distances, and examines how much cooperation can be sustained as the circle grows in size. He finds that small and large worlds can sustain greater trust than their intermediate counterparts. In small worlds, partners to a transaction are likely to know third parties in common, and thus are able to share information about defectors. In large worlds, developing a legal system becomes economical. In contrast, our model focuses on the case where information sharing about behavior within bilateral relationships is not possible.

[12] In using the term "identity" we follow Akerlof and Kranton [1], [2]. We differ, however, in the form that identity takes. In their formulation, group "identity" enters into individuals' utility functions. These identities and the concomitant "prescriptions" for behavior that they imply result in individual and group sanctions for violators of the appropriate group "code of conduct."

inefficiently long, because the institutions that support cooperation must entail costs of starting new relationships. In contrast, the social hierarchies we construct are "impersonal" in the sense that the actual agents in the hierarchy can change but they inherit the incentives of their rank and thus fully efficient exchange can be sustained. This is a distinct advantage of social hierarchies that has not to our knowledge been explored in the economics literature.[13] Specific identity investment also has parallels in an important literature, beginning with Kreps [31], on the role of "corporate culture". Culture can create value for the firm through variety of means, including reducing costs of coordination and communication and improving commitment by managers.[14] Homogeneity within firms can happen through selection and through "learning" and indoctrination (Van den Steen [48]). Indoctrination and learning to communicate within the firm can be thought of as specific investments in the firm culture or identity. Thus, though we abstract from the potential productive roles of identity investment, our discussion of the relative robustness of hierarchies and identity investment equilibria can shed light on the relative robustness of trust-enhancing aspects of corporate culture as well.

Thus, cooperative equilibria based upon the existence of barriers to form new relationships occupy a prominent role in theories of trust.[15] An important focus of our study is to compare the robustness properties of such identity investment equilibria in environments where some groups adopt social hierarchies and there may be insurgent groups that lower barriers to entry entirely.[16] Our analysis also differs from much of the existing literature on trust in its focus on endogenous group formation, group size, hierarchical structures, and the problems associated with increasing population size.

3 The Model

The game takes place over an infinite horizon, with periods indexed by $t = 1, .., \infty$. There is a stationary population of N players, with N even. Each individual survives with independent probability δ every period. When a player dies or leaves another player inherits his index with a null history. Trade among individuals takes place within J "markets" which can be thought of as actual geographical marketplaces, virtual communities or simply groups of people who recognize each others' affiliation. At the beginning of each period, individuals select their market. For simplicity we assume that newly born individuals select markets only after returning members

[13] The key distinction between an agents rank and their personal identity naturally has a long tradition in sociology. In particular, we build on and further work, at least as early as White [51], on mobility in hierarchical organizations and

"chains of vacancies created by openings at higher ranks of hierarchies. (see also Gibbons [17]).

[14] Hermalin [23] provides a very valuable overview.

[15] A notable exception to the focus on entry barriers to sustain trust is work by Lindsey, Polak and Zeckhauser [34], who incorporate the notion of itinerant temptations into their study of long-term bilateral, exclusive relationships between individuals where there is "free love": no barriers to a new start. Existing relationships gain value the longer they exist. This makes them robust to break-ups. However, this occurs for different reasons in our model: in the social hierarchies we construct, senior agents engender more trust.

[16] In comparing the competitiveness of alternative forms of social organization, our paper has natural links to research in organizational ecology (see e.g. Hannan, Polo and Carroll [21].)

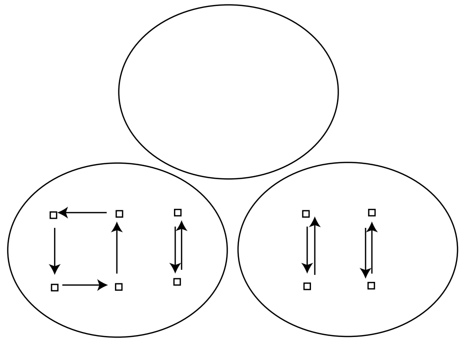


Figure 1: 3 markets, including 1 "potential" market. Arrows denote the direction of principal-agent relationships. Individuals can act as agents to one and principals to a different trading partner.

have chosen and that N is so large that moving to a new market is always an option. Markets may have capacity constraints. We allow the capacity constraints to be specified as part of an equilibrium.

Next, we consider the interaction of players in a market. Individuals engage in transactions in pairs. At the beginning of each period they are randomly matched to each other. Each pair has a principal and an agent. The randomization ensures that each individual is an agent to no more than one principal and a principal to no more than one agent. A market will therefore have N/J trust games taking place. As Figure 1 illustrates, an individual will typically be a principal to one individual and an agent to another.

After being matched, the principal chooses how much to trust the agent, offering a scale of trade $\lambda \in [0, 1]$ to the agent. Then, if the agent works, payoffs are $(\lambda R, \lambda w)$ to the principal and agent, respectively. If an agent shirks, payoffs are $(-\lambda r, \lambda s)$. We assume $w < s$, $r > 0$ and $R + w > s - r$, so shirking is inefficient.

Players observe personal bilateral trade histories from birth to the current period, but not the trades of other players. Individuals hence keep track of the actions of a given partner in all previous direct interactions, if any, either as an agent (whether they worked or shirked), as a principal (the scale of trust), and of their own actions.

We assume that markets keep record of individuals' continuous attendance, which we refer to as an "attendance spell." This information is costlessly observable to market participants. It is updated every period after attendance choices are made and before trade takes place. Thus individuals can observe how many periods a potential trading partner has continuously attended a given market (i.e. his seniority), even if they have not previously traded. Further, the record sharing is *local*: each market has no record of the other markets an individual attended before joining.

A (period) trading strategy is a mapping from the set of all possible bilateral trading histories and attendance records to the market to attend today; as principal how much to trust each potential agent, and as agent to shirk or work for each potential principal.

A variety of possible equilibrium concepts can be employed to solve this extensive-form game of imperfect information. We will restrict attention to Sequential Equilibria (SE) as this imposes beliefs where each agent's beliefs are such that the subjective probability of meeting trustworthy individuals off of the equilibrium path is maximal. Thus, bilateral sanctions are weakest and cooperation most difficult to sustain.[17]

4 Benchmarks

In this section, we consider potential classes of equilibria that are "institution-free" in the sense that individuals condition only on their bilateral trading history when determining trust. Not surprisingly, providing individuals are extremely patient (long-lived), it is possible to sustain cooperation, even with large populations and alternative trading partners. In the *peddler's equilibrium*, traders randomize uniformly over markets in every period. Within each pair, agents work and principals maximally trust unless either individual deviated in past play, in which case both partners revert to shirking if trusted and offering zero trust. In fact, in such an equilibrium, players perceive it as equally likely to be matched with all other individuals in the population *as if* there were no markets at all.

Proposition 4.1. *For any δ, N, R, w and s satisfying our basic assumptions, a peddler's equilibrium exists if and only if*

$$s - w \leq \frac{\delta^2}{1-\delta^2}\frac{1}{N-1}(R+w). \tag{1}$$

Proof: A formal proof is available in the appendix. □

(7) is the "no-shirking" incentive constraint. The right hand side is the value of a cooperative relationship: the stream of profit that would be lost with a loss of reciprocal trust. $\frac{\delta^2}{1-\delta^2}$ weights such stream by the probability that both individuals are alive and $\frac{1}{N-1}$ is the probability that the player matches the partner again as an agent or as a principal, conditional on both partners being alive.

As (7) reveals, population size reduces the probability of repeated interaction and thus the effectiveness of the sanction on cheaters. For a given level of patience, increasing population will thus lead to a failure of cooperation in the peddler's equilibrium. Note also that modifying the scale of trade does not affect this constraint: if all trades take place at a smaller or larger scale, the key ratio $(s-w)/(R+w)$ remains unchanged, and cooperation will fail as N rises.

It may seem plausible that individuals could sustain cooperation in large populations by becoming "loyal" to a sub-group: returning to the same market every period and trusting all

[17] For example, if we instead used a weak Perfect Bayesian Equilibrium, in what follows, we could support cooperation more easily by allowing beliefs such that individuals that have experienced a defection defect themselves even against new partners. This transmission will be a best response given that defected against agents believe that their next partner will also defect, and thus that the contagion is sufficiently widespread. Naturally, this imposes an additional cost to any initial defection and helps sustain cooperation.

agents that also do so. However, this is not the case: if individuals follow pure strategies in selecting markets, then one individual can avoid another forever by leaving for a new market after cheating, avoiding any sanction as a result. Thus equilibria based purely on “loyalty” cannot typically be sustained.

5 Communities with Institutions

The previous section revealed the failure of bilateral relationships to maintain cooperation with large populations. In this section, we consider alternative “institutions” that might allow cooperation for arbitrary population sizes. We begin by analyzing equilibria based on specific investments, and we then turn to seniority-based hierarchies.

5.1 Investments in Identity

One way of sustaining cooperation as populations rise is to require individuals to make *specific* investments in a cultural identity that marks them as members of a market. Such investment can be either publicly observable or simply required to join existing markets. We examine symmetric equilibria where all markets select the same level of required investment, denoted m.

An **identity investment equilibrium** is an SE of our game with the following properties: markets are capacity constrained and, for simplicity, symmetric. Let $N_m \equiv N/J$ denote the number of individuals in each of the J markets.

At birth each individual chooses a market that has a vacancy and makes the specific investment. He then returns to that same market unless he deviated from the above strategies in past play in which case he may possibly select a new market as his new “home.” A new investment is due every time an individual joins a new market.

Within each market, agents work and principals maximally trust if none of their previous trading partners deviated from full cooperation in past play. Further, in each bilateral relationship, agents continue to cooperate unless either trading partner deviated with the other in past play in which case both partners revert to offering zero trust and shirking if trusted.

We now specify the conditions for existence of an identity investment equilibrium. If the agent shirks, he faces the following choice: endure the sanction of no future trade with the principal he has cheated, whom he is likely to run into if he continues to attend the current market; or else pay the investment cost elsewhere and start anew. To simplify the analysis we restrict attention to SE in which: (1) individual attendance choices depend only on how many untrustworthy individuals (or “enemies”) x were present in the previous attendance spell (henceforth the “state”) and (2) individual trade choices depend only on x and on bilateral trading histories. The uncertainty in the game is induced by the facts that, (i) at the beginning of each trading period, individuals die with some positive probability, and (ii) agents are randomly assigned to principals. The transition probabilities depend on δ and on individuals’ actions. Conditional on the others cooperating, the individual’s problem is that of finding an optimal plan mapping realized states into the following choices: as an agent, to either work-and-return, cheat-and-return or cheat-and-leave, and as a principal, how much to trust agents. In the identity investment equilibrium, the strategies always specify full trust by principals on the equilibrium path. Under standard regularity conditions which are met in this formulation (i.e. finite state space, finite action space, "memoryless" transition probabilities and discounting), a solution to this problem always exists and is unique, up to eventual indifferences.[18] The challenge becomes to find conditions on our primitives under which cooperation is sustained in the resultant dynamic program.

Let $W(x, W^L(m))$ denote the value function for an individual at the beginning of a trading period, before choosing a market to attend. This continuation payoff depends on an individual's history of cheating behavior against all agents present in the previous attendance spell, summarized by state x, and on the present value of leaving for a new market that sustains cooperation: $W^L(m) \equiv (R+w)/(1-\delta) - m$. Let $p(i,x)$ denote the probability that i out of x enemies survive conditional on the agent surviving and $\alpha(i, N_m)$ denote the probability of being matched with a trustworthy individual when there are i enemies around.

Proposition 5.1. *For any δ, N, R, w, and s satisfying our basic assumptions:*

(i.) an identity investment equilibrium with $N_m = 2$ and $J = N/2$ active markets exists if and only if

$$s - w \leq \frac{\delta^2}{1-\delta^2}(R+w) \qquad \text{and} \qquad m \geq \underline{m} \equiv \frac{s-w}{\delta}$$

hold.

(ii.) An equilibrium also exists with $N_m > 2$ and $J = N/N_m$ active markets if and only if

$$s - w \leq \frac{\delta^2}{1-\delta^2}\frac{1}{N_m - 1}(R+w), \qquad (IC_{AR})$$

and $m \geq \underline{m}(N_m)$ where $\underline{m}(N_m)$ uniquely solves:

$$s - w = \delta\left(\frac{w+R}{1-\delta} - W^C(1, W^L(m))\right), \qquad (IC_{AL})$$

where $W^C(0) \equiv (r+w)/(1-\delta)$ and $W^C(x, W^L(m))$ is defined by the following system of equations:

$$W^C(x, W^L) = \max\{W^L, \sum_{i=0}^{N_m-1} p(i,x)[\alpha(i,N_m)[(s+R)+\delta W^C(i+1, W^L)]+(1-\alpha(i,N_m))\delta W^C(i, W^L)]\}. \qquad (2)$$

Proof: (i.) Ignoring the option to leave, the first condition ensures that agents have no incentive to cheat and return to the home market. $m \geq \underline{m}$ guarantees that agents have no incentive to

[18] We assume that individuals always cooperate whenever indifferent.

cheat and leave for a new market. We are left to verify that the principal also prefers to return after being cheated (so that he is there to provide the sanction). Since both individuals see the same value of returning to the home market it is sufficient to restrict attention to equilibrium profiles where the principal always returns. A formal proof of part (ii.) is available in the appendix. □

When $N_m = 2$ (i.e. with partnerships) the two conditions have a straightforward interpretation. The first inequality guarantees that the threat of future retaliation is sufficient to induce cooperation with one's partner, absent the option to leave. The second guarantees that cheating and leaving is not a viable alternative. For δ and m high enough an equilibrium always exists. The efficient membership fee must rise with temptation to cheat and fall with patience. Part (ii.) is slightly more complex as it accounts for the possibility that individuals may cheat-and-return for a number of periods before leaving off the equilibrium path. This is because with $N_m > 2$ there is a chance to meet a trustworthy individual in the continuation game that follows a cheat-and-return defection. The optimal "exit" strategy **can** involve a number of cheat-and-return defections before finally quitting the group.

We now focus on the role of group size on existence and efficiency.

Proposition 5.2. *Identity investment equilibria are most likely to exist with two-agent partnerships. Furthermore two-agent partnerships maximize the total surplus generated in the economy provided that m is set at the lowest level compatible with incentive compatibility $\underline{m}(N_m)$.*

Proof: A formal proof is available in the appendix. □

The conditional probability of meeting an untrustworthy individual in the continuation game starting after cheating-and-returning to the home market decreases with N_m. Since this reduces the sanction associated to cheating it follows that smaller subgroups can be sustained at lower levels of patience. In fact the lowest level of patience required is for subgroups of two individuals.

By scaling down the "small numbers" requirement at the group level (as opposed to at the population level), identity investments can help restore cooperation when condition (7) fails. In fact as long as the number of (endogenous) markets grows proportionally, the number of individuals can grow arbitrarily large without upsetting incentives. Furthermore, since the payoff from cheating-and-returning can exceed that of leaving right after cheating, smaller groups require weakly lower membership barriers. Therefore smaller groups are more efficient as well. Hence absent technological reasons for group production (or diversity), "marriages" or exclusive relationships are most desirable.

Note that the scale of trade plays no role in the above analysis. Reducing λ below 1 does not make it easier to support this equilibrium, since this just scales down payoffs from both cheating and cooperating.

An unattractive feature of the identity investment equilibria is that identity investments act as a barrier to entry by members of other markets, not to retain members of one's own.

Implicitly, the strategies used by members of one market are constructed in order to sustain the cohesiveness of another. If the members of even one market fail to impose barriers to entry, cooperation will fail in other markets.[19]

We now proceed to investigate to what extent strategies that make use of information on attendance can substitute for costly investments. In particular we investigate profiles whereby individuals discriminate among trading partners, conditioning the scale of trade on seniority.

5.2 Purgatorial Equilibria

In this section we introduce the concept of "seniority" by making use of market attendance records. We construct an equilibrium in which players are partially or totally excluded from trade in their first few periods of attendance at a new market.

A **purgatorial equilibrium** is an identity investment equilibrium with the following modifications. There are no fixed membership fees ($m = 0$). Within each market, agents work and principals trust trading partners that did not deviate in past play according to a scale which depends on attendance. Individuals whose current attendance spell is at least t^* periods long are maximally trusted. Agents who have been present for less than t^* periods are instead trusted at some scale λ^* in $[0, 1)$.

For simplicity, in what follows we focus on the case where purgatory can be only one period. At the beginning of each period, $(1-\delta)N_m$ individuals are expected to die so there will be on average $(1-\delta)N_m$ new members. The equilibrium value for an agent who has completed purgatory is hence:

$$W(\lambda^*) \equiv \frac{1}{1-\delta}\left(w + \delta R + (1-\delta)\lambda^* R\right).$$

In contrast, a newcomer to a market expects on average $(1-\delta)N_m/(1-\delta^{N_m}) - 1$ other agents to be in purgatory at the same time. That is, the total expected vacancies **other than that occupied by the agent**. The value for an individual beginning purgatory is thus:

$$W^L(\lambda^*) \equiv \frac{(1-\delta)N_m - 1 + \delta^{N_m}}{(1-\delta^{N_m})(N_m - 1)}(\lambda^* w + \lambda^* R) + \frac{N_m(\delta - \delta^{N_m})}{(1-\delta^{N_m})(N_m - 1)}(\lambda^* w + R) + \delta W(\lambda^*).^{20}$$

[19] An interesting variant of an identity investment equilibrium is suggested by Marin and Schnitzer [35]. They suggest that a reason for the reduced use of cash in Russia, was that partnership-specific bartered goods reduce the incentive to cheat and leave over settling in cash. In this way, the loss from selling bartered goods in the market relative to cash can also be thought of as an identity-specific investment.

It is possible to find other reasons through which costly identity investments actually serve a group's interests. For instance, in a world with heterogeneous individuals and technological reasons for screening, investments can be designed in order to attract those individuals from which the group would benefit the most. Therefore acting as a barrier to (unwanted) members of other markets can actually serve the goal of the market. Criminal organizations typically design and rely upon both entry and *exit* barriers to sustain cooperation in environments in which the legal ordering is weak but trust is needed to conduct business. Investment costs may also be interpreted as natural barriers to entry, such as cultural (e.g. language, gender, race, religion) or physical (e.g. geographical location). A model with correlated costs within different groups of individuals could for instance explain segregation as a result of individuals trying to minimize their cost of entry.

[20] The $1/(1-\delta^{N_m})$ factor accounts for the fact that communities with *at least* one vacancy, have on average more vacancies. Further details on how we constructed the payoffs are relegated in the appendix. In brief, the observation "the community has at least one vacancy" is informative about the realization of the random variable

Proposition 5.3. *Suppose that for some δ, N, r, R and s an Identity Investment equilibrium exists with J active markets, then a Purgatorial equilibrium exists with the same number of markets only if there is a λ^* in $[0,1]$ such that*

$$\underline{m}(N_m) \leq W(\lambda^*) - W^L(\lambda^*), \tag{3}$$

where $\underline{m}(N_m)$ denotes the minimum level of investment that supports the Identity Investment equilibrium.

Proof: See the appendix for a formal proof. □

Equation (3) is stated as a non-primitive condition; however, it is straightforward to write the right-hand side in terms of primitives. The left-hand side was defined in Proposition 5.1. For the case where $N_m = 2$, $\underline{m}(N_m) = (s - w)/\delta$, and the equation reduces to

$$\frac{s-w}{\delta} \leq (1-\lambda^*)w + \frac{(1-\lambda^*)\delta(1-\delta)R}{1+\delta}$$

Equilibrium calls for reduced trade with newcomers to provide a cost of starting over in a new market. The proposition says that the gap in continuation payoffs should be large enough.[21] We interpret the gap in equilibrium payoffs as the capital value of seniority. That is, how much individuals would be willing to pay to "skip" purgatory. Since the newcomers' payoff is more sensitive to changes in λ^*, the gap decreases with λ^* provided that R is low enough.[22] Lower trust levels on one hand reduce efficiency whereas on the other relax incentive constraints.

One key difference between the identity investment equilibrium with membership fees and the purgatorial equilibrium concerns the role of group size. The fact that newcomers are relatively less likely to be matched with other newcomers (since they will not be matched to themselves) gives them a comparative advantage over seniors since it is easier for them to secure the more trustworthy agents and hence realize the full value R. In fact when $R = 0$ there is no advantage for newcomers: the gap between returning members' and newcomers' payoffs is simply $(1-\lambda^*)w$. For positive values of R the value of seniority is strictly lower than $(1-\lambda^*)w$, possibly negative. This poses a challenge on existence. Bigger markets, by diluting the newcomers' advantage, increase the gap and can thus potentially support equilibria with higher scales of trade. The following proposition summarizes the above claims.

Proposition 5.4. *The value of seniority increases with market size and decreases with R.*

Proof: The result can be established algebraically. □

of interest for a newcomer, i.e. the total number of individuals who "died" (denoted $\tilde{D}$) in that the event "no one died" (which occurs with probability (δ^{N_m})) is certain not to have occurred. Hence $\tilde{D} = (1-\delta)N_m/(1-\delta^{N_m})$.

[21] The reason why this condition is not always sufficient to warrant existence is analogous to that of the previous section. Individuals might defray the opportunity cost of starting over by cheating for a number of periods before quitting the market. We provide the necessary and sufficient condition in appendix.

[22] The claim that $W - W^L$ decreases with λ^* is true if and only if $\frac{R}{w} < \frac{(1-\delta^{N_m})(N_m-1)}{(1-\delta^{N_m})-(1-\delta)(1+\delta^{N_m}(N_m-1))}$. (This can be easily established algebraically.) The reason why the qualifier is needed follows the discussion of the comparative advantage provided hereafter.

Since the left hand side of (3) increases with N_m and there can be no benefit in returning to a market with one untrustworty individual, purgatorial equilibria are most likely to exist with $N_m = 2$ and $R = 0$. When $R > 0$, larger groups reduce the juniors' comparative advantage (the payoff gap between senior members and newcomers increases with N_m). Thus bigger groups can potentially support equilibria with higher scales of trade λ^* and can thus be more efficient, so that there is a tradeoff between existence and efficiency.

A growing body of evidence suggests that poor societies with heterogeneous social and cultural identities provide fewer public goods and face reduced growth trajectories (eg Alesina and La Ferrara [3]). Less is known about the factors that may lead societies to maintain cultural and ethnic differentiation.[23] This analysis suggests that, absent technological factors such as gains from specialization or taste for variety, exogenous dimensions of difference allow identity-specific investments that can actually be relatively efficient at facilitating trust, particularly among smaller subgroups. In contrast, approaches to sustaining trust that rely on reduced trust for newcomers are less efficient. The inefficiency declines with group size, so if some other exogenous force favors larger group sizes, the relative inefficiency of purgatorial equilibrium becomes less important.

5.3 Hierarchical Equilibria

Like the identity investment equilibrium, in the purgatorial equilibrium each market's enforcement of purgatory requires individuals to refrain from trade purely for the purpose of supporting social conventions that help sustain cooperation. We now explore the extent to which this feature can be relaxed.

We consider a "hierarchical structure" for each market to be a number of levels, L, and a vector of numbers, $(\gamma_1, \gamma_2, ...\gamma_L)$, where γ_l is the fraction of individuals at level l. For simplicity we consider only cases where the number of agents at each level l per market $\frac{\gamma_l N}{J}$ is an integer. A simple probabilistic advancement process takes place at the beginning of each period. Empty places further up the hierarchy are randomly assigned to individuals at lower levels. Priority is given to more senior members in that, empty places at some level l are firstly randomly allocated to individuals at level $l-1$, then as necessary to individuals at level $l-2$ and so on. Starting the process from the top of the hierarchy, promotions create new vacancies at lower levels which are successively filled according to the same procedure. Finally, newly born individuals fill in the remaining places. Note that, since individuals die with independent probability, both existing members and newcomers can potentially "skip" one or more levels, possibly entering the top level directly.

Let $\lambda_l \in [0, 1]$ denote the equilibrium trustworthiness of an agent at level l. A **hierarchical equilibrium** is a SE of our game that satisfies the following properties. Markets are symmetric

[23] Beyond the theoretical literature on identity investments already discussed, other possibilities that have been noted that may account for such continued differences include altruistic cultural transmission (eg Bisin and Verdier [7]), aspiration traps (Ray [43]) and coalitions competing over public goods (eg Posner [41], Esteban and Ray [12]).

and capacity constrained. Within each market, agents work and principals trust level l agents at scale λ_l, unless their trading partners deviated in past play, in which case they revert to shirking if trusted and offering zero trust. The scales of trade are monotonic in seniority, that is: $l' > l$ implies $\lambda_{l'} \geq \lambda_l$. $\lambda_L > 0$. At birth each individual chooses a market that has a vacancy. He then returns to that same market unless he deviated from the above strategies in past play in which case he may possibly select a new market as his new "home." A **maximal-trust hierarchical equilibrium** is a hierarchical equilibrium in which individuals are either fully trusted or else trusted up to the point where their incentive constraints bind.

Consider how such an equilibrium works. The increasing scale of trade together with our advancement process imply that an individual's value for being in a market is increasing in his seniority. Individuals lower in the hierarchy have less to lose by starting over in a new market and hence can be less trustworthy. Furthermore, unlike the purgatorial equilibrium, advancements are probabilistic. Hence individuals at the next-to-highest level have a lower continuation value than those at the highest level. Thus, in contrast to a purgatorial equilibrium, it is possible that in each potential pairing, agents are trusted as far as they can be.

Consider now a more formal analysis. Our interest is in how changes in the primitives of the model and in the seniority structure affect both existence and efficiency of maximal-trust equilibria. The equilibrium (period) payoff for an agent at seniority level l is:

$$\Pi(l) \equiv \frac{1}{N_m - 1}\left(N_m \sum_{l'=1}^{L} \gamma_{l'} \left[\lambda_{l'} R + \lambda_l w\right] - \lambda_l (R + w)\right),$$

which rises with seniority. Let $W(B, l)$ denote the value function of an individual with seniority level l and history of cheating behavior in the home market summarized by a list B that keeps track of the seniority levels of those "enemies" that were present in the previous attendance spell. Let $W(\emptyset, 0)$ denote the value of a new arrival to a group and $\beta_{l,l'}$ the probability that a type l transitions to level l' conditional on surviving. Then for an individual with no history of cheating, the value is:

$$W(\emptyset, l) \equiv \sum_{l'=1}^{L} \beta_{l,l'} \left(\Pi(l') + \delta W(\emptyset, l')\right).$$

Now consider the constraints that must be satisfied for the specified strategies to be best responses. First, all individuals should prefer cooperation to cheating and leaving. That is, for each l we should have that

$$\lambda_l (s - w) \leq \delta \left(W(\emptyset, l) - W(\emptyset, 0)\right). \qquad (IC_{AL})$$

Next, individuals should be deterred from returning to the home market after cheating. To simplify the analysis, we look for equilibria in which the value of starting over in a new market exceeds that of returning to the home market after cheating. This guarantees both that cheaters leave and that individuals that have been cheated always return.[24] Since the greatest temptation to return to the home market after cheating will be faced by an agent of highest level (L) after cheating a principal of the lowest level (1), the relevant constraint is:

$$W(\emptyset,0) \geq \delta \sum_{l=1}^{L} \tilde{\beta}_{1,l}(L)[\Pi^s(L,l) + \delta W(\emptyset,0)] + (1-\delta)W(\emptyset,L) \qquad (IC_{AR})$$

where $\tilde{\beta}_{1,l}(L)$ is the probability that an individual of level 1 transitions to level l conditional both on surviving and on an individual of level L returning and

$$\Pi^s\left(l,l'\right) = \frac{1}{N_m - 1}\left(\begin{array}{c} N_m \sum_{h=1}^{L} \gamma_h \left(\lambda_h R + \lambda_l s\right) - \lambda_l \left(R+s\right) \\ -\left[\lambda_{l'} R + \lambda_l s\right] \end{array}\right)$$

is the per period payoff of an agent of seniority level l shirking, having cheated a principal of current level l' in the previous period.[25]

For simplicity, in what follows we focus on maximal-trust equilibria with two-level hierarchies.[26]

Proposition 5.5. *(i.) [Existence of Maximal-trust Equilibrium] For any $\gamma_1' > 0$, one can always find some $\delta < 1$ and $(s-w) > 0$ small enough such that a Maximal-trust Hierarchical Equilibrium exists with $\gamma_1 = \gamma_1'$.*

(ii.) [Role of Maximal Trust Restriction] If a Hierarchical Equilibrium exists with $\gamma_1 = \gamma_1'$ for a given set of exogenous parameters then a Maximal-trust Hierarchical Equilibrium also exists if and only if

$$\frac{1-\beta_{1,2}'}{1-\delta(1-\beta_{1,2}')} \geq \frac{s-w}{\delta}\left[w - \frac{R}{N_m - 1}\right]^{-1}, \qquad (4)$$

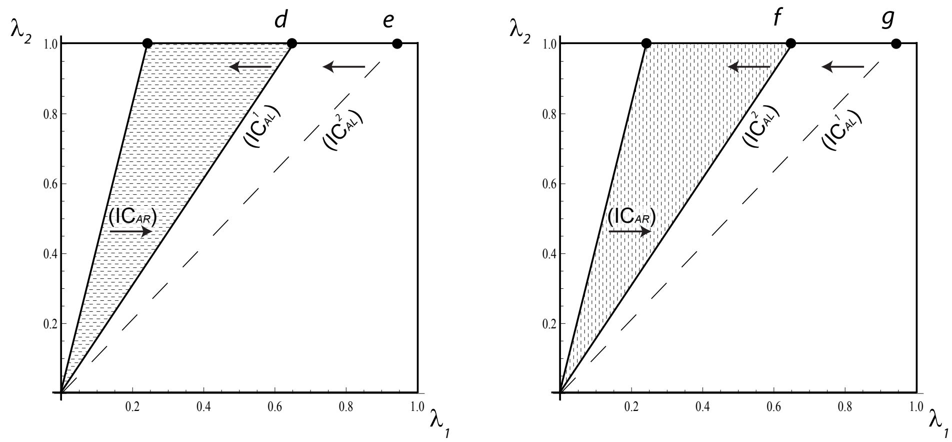
and is characterized by trust levels $\lambda_2 = 1$ and $\lambda_1 < 1$ satisfying the juniors' "cheat-and-leave" constraint (IC_{AL}) with equality.

(iii.) [Efficiency of Maximal-trust Equilibrium] The Maximal-trust Equilibrium maximizes the total value of trade over the set of hierarchical equilibria.

[24] In this setup, individuals with a lower seniority level have a higher opportunity cost of returning to a market with some untrustworthy individuals. Thus the fact that agents find optimal to return upon cheating (conditional on the principal returning) does not necessarily imply that the principals will indeed find it optimal to return, as it did in section 5.1.

[25] If an equilibrium exists in which agents leave in $t+1$ after cheating to a junior member in t then in equilibrium agents must also find optimal to leave in $t+2$ upon returning in $t+1$ when supposed to leave, conditional on the enemy surviving, hence the $\delta W(\emptyset, 0)$ term on the rhs.

[26] We conjecture that two-level hierarchies can accomplish most of what could be implemented with additional levels. However, the addition of more levels may allow for fine-tuning of transition probabilities, so that a three-level hierarchy can potentially widen the range of parameters in which a maximal-trust equilibrium exists.



(a) ICs Satisfied, (4) holds

(b) ICs Satisfied, (4) fails

Figure 2: Incentive Constraints

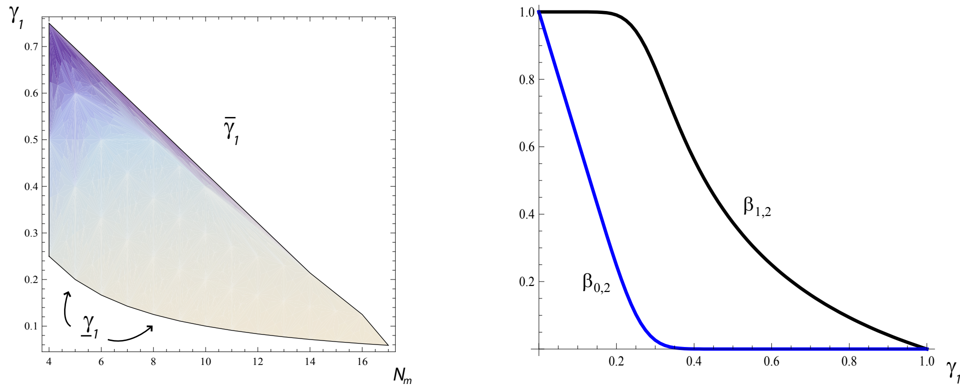
Proof: To prove i., it is useful to rewrite the incentive constraints in terms of our primitives as follows:

$$\lambda_2 \frac{s-w}{\delta} \leq \Delta a, \qquad \lambda_1 \frac{s-w}{\delta} \leq \Delta b, \qquad (IC'_{AL})$$

$$\Delta a \leq \frac{\delta}{1-\delta^2} \Delta^s, \qquad (IC'_{AR})$$

where $a \equiv (1-\beta_{0,2})/(1-\delta(1-\beta_{1,2}))$, $b \equiv (\beta_{1,2}-\beta_{0,2})/(1-\delta(1-\beta_{1,2}))$, $\Delta \equiv \Pi(2)-\Pi(1) = (\lambda_2-\lambda_1)(w-R/(N_m-1))$, and $\Delta^s \equiv \Pi(2)-(\tilde{\beta}_{1,2}\Pi^s(2,2)+(1-\tilde{\beta}_{1,2})\Pi^s(2,1))$. The right hand side of both the seniors' and juniors' "cheat and leave" constraint (IC'_{AL}) is the opportunity cost of starting afresh in a new market (i.e. the "gap" in continuation values between returning members and newcomers to a market). It is proportional to the difference in the per-period payoffs Δ, which in turn is proportional and increases with $\lambda_2-\lambda_1$. The right hand side of (IC'_{AR}) is the lost stream of profits $W(\emptyset, L) - W(\{1\}, L)$. It is proportional to the difference between the seniors' per-period payoff on the equilibrium path and the expected payoff of one period of shirking in a market that has one untrustworthy individual. Note that the incentive constraints are weighed sums of payoffs and are linear in the trust levels. A hierarchical equilibrium exists if and only if the constraints define a non-empty region in the (λ_1, λ_2)-plane. To prove Proposition 5.5, we need to show that any time this region is non-empty, there is also a non-empty region where $\lambda_2 = 1$ (required for seniors to be trusted as far as possible), the junior's (IC_{AL}) binds, and the senior's (IC_{AL}) is satisfied. The point is illustrated intuitively in Figure 2. (4) guarantees that the junior's (IC_{AL}) constraint lies to the left of the senior's (IC_{AL}) (in the Figure , point d is to the left of e), which establishes the result. Parts ii. and iii. can be easily verified using (IC'_{AR}) and (IC'_{AL}). □

Together (i.), (ii.), and (iii.) imply that focusing on maximal-trust equilibria comes at no cost in terms of efficiency and, when (4) holds, no additional restrictions on existence either.



(a) Existence Region (b) Transition Probabilities

Figure 3: $w = r = 1, s = 1.1, \delta = 9/10$.

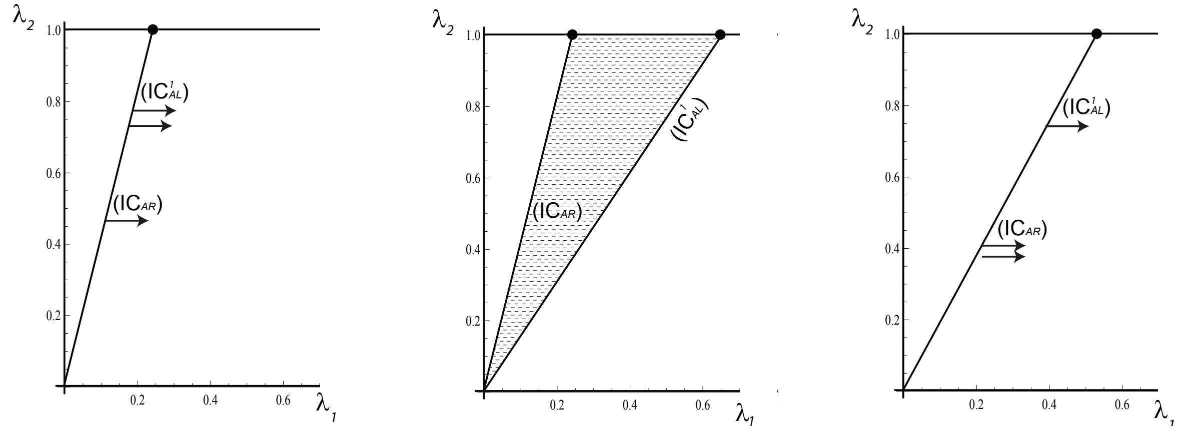
Thus, when (4) holds the added robustness properties can be imposed without sacrifice. (4) is satisfied when the gain to shirking is small enough relative to the discount factor. When (4) fails, increasing the level of trust granted to junior members eventually leads *seniors* to defect and leave.[27] Note that the maximal-trust equilibria is necessarily located at the top-right corner of the existence region depicted in Figure 2a as otherwise one could increase either both or at least one trust level without upsetting the incentives. Thus in equilibrium juniors are indifferent between cooperating and shirking whereas the seniors' incentive constraints are slack.

How does group structure –captured by pairs of group size and proportion of juniors (N_m, γ_1)– affect existence and efficiency of maximal-trust equilibria? Figure 3a shows the existence region for an arbitrary set of parameters that satisfy our basic assumptions. The shaded area corresponds to seniority structures where a maximal-trust equilibrium exists. The brighter the texture, the more efficient the structure.

We begin by holding fixed group size, and studying the role of group structure. Consider first the issue of existence. Cooperative hierarchies must be those where upward social mobility (that is the degree to which an individual's seniority status can change) is neither too low nor too high. Intuitively this is due to the fact that the "asset value" of being a returning junior (as opposed to a newcomer) depends on the *difference* between the newcomers' and the juniors' transitional probabilities to higher levels (Figure 3b). This difference, which is hump shaped as the proportion of juniors increases, drives the gap in continuation values that also provides incentives to return to a market or start anew. If climbing to higher levels of a hierarchy is excessively easy or excessively hard then being a returning member does not confer any relative advantage: the asset value is too low to induce loyalty.

Although the juniors' incentives to cooperate firstly increase and then decrease with γ_1, the existence region is not necessarily a convex subset of $(0, 1)$ in γ_1, although Figure 3 illustrates

[27]Since seniors are trusted more in equilibrium, their present gains from shirking are higher and increase with $(s - w)$. Raising λ_1 makes shirking more attractive in that it reduces the opportunity cost of quitting for a new market. Thus it can be the case that a marginal increase in λ_1 triggers the seniors' defection (point f in Figure 2b).



(a) $\gamma_1 = \gamma_1' > 0$ (b) $0 < \gamma_1' < \gamma_1 < \gamma_1'' < 1$ (c) $\gamma_1 = \gamma_1'' < 1$

Figure 4

a case where convexity holds. The potential source of discontinuity is illustrated intuitively by means of Figure 4. The arrows point in the direction in which the constraints move with γ_1. Raising the proportion of juniors γ_1 in the structure makes it *harder* to satisfy the seniors' (IC_{AR}), since seniors have relatively more to lose from a "cheat and leave" defection relative to "cheat and return". At the same time both the juniors' constraints are relaxed for relatively low values of γ_1. If both constraints "travel" in the same direction, then without imposing further restrictions, one cannot exclude the possibility that the constraints "overtake" one another for different levels of γ_1, creating a gap.

Now consider the issue of efficiency. The total (per period) surplus in equilibrium is

$$W \equiv 2N_m(R+w)[1-\gamma_1(1-\lambda_1^*)] \tag{5}$$

where λ_1^* is the highest scale of trade that preserves the juniors' incentives to cooperate. Differentiating yields:

$$\frac{dW/2N_m}{d\gamma_1} = \underbrace{-(1-\lambda_1^*)(R+w)}_{\text{Mismatch effect (-)}} + \overbrace{\frac{d\lambda_1^*}{d\gamma_1}\gamma_1(R+w)}^{\text{Incentive effect (+/-)}} \tag{6}$$

Consider the case where γ_1 is below the level that maximizes the gap in transition probabilities. Then (11) shows that a marginal increase in the proportion of juniors has two effects. First, it exacerbates the per agent welfare losses due to "mismatches," as principals withhold trust to more junior members. Second, however, as discussed above, over this domain, raising the proportion of juniors also relaxes the incentive constraints, thereby allowing for higher levels of trust if the equilibrium already exists, and also potentially permitting existence where it does not.

In per agent welfare terms, the losses due to the mismatch effect will dominate the gains from the incentive effect for low levels of γ_1.[28] This implies that:

Proposition 5.6. *Efficient seniority structures are characterized by the smallest proportion of juniors γ_1 that guarantees existence of a maximal-trust equilibrium.*

Finally, we consider the effect of N_m on existence. Although in the illustration in Figure 3 the existence region is convex in N_m we have not established whether convexity is a general property of the existence region. To see why this issue is subtle consider how N_m affects incentives one constraint at a time. First, bigger groups make it harder to sustain cooperation for the usual reasoning: the temptation to cheat-and-return is higher (Δ^s increases). Ceteris paribus, the existence region "implodes" with N_m (i.e., if $N'_m > N_m$ then $\underline{\gamma}_1 < \underline{\gamma}'_1 < \overline{\gamma}'_1 < \overline{\gamma}_1$). However larger markets also enable steeper incentives to seniority. Conditional on surviving, juniors have a relatively higher chance of being matched with a senior agent. This relative advantage decreases with N_m thus increasing the gap $\Pi(2) - \Pi(1)$ as long as $R > 0$. A bigger gap, ceteris paribus, relaxes the (IC_{AL}) constraint and has the opposite impact: bigger groups make it easier to sustain cooperation.

It follows that overall the boundaries of the existence region need not be monotone (unlike the ones illustrated in Figure 4a) and the region need not be convex. However since the marginal impact of a unit increase of the group population on $\Pi(2) - \Pi(1)$ goes to zero as N_m goes to infinity, eventually market expansion hinders cooperation.[29]

To summarize, a hierarchical structure that is restricted so that all agents are either fully trusted or trusted to the full extent that they are trustworthy can sustain cooperation in equilibrium as long as upward social mobility (that is the degree to which an individual's social status can change) is set so that it is neither too low nor too high. Furthermore efficient social structures feature lower levels that are as small as possible while preserving some randomness as to whether lower-level agents are promoted. Much as in the previous section, to the extent that cooperation can be sustained, bigger groups can potentially increase the overall surplus by allowing for smaller lower levels.

5.4 Choice-Based Hierarchical Equilibrium

Up until this point, we have assumed that individuals can choose between markets, but once inside a market, matching is random. In some settings and for some kinds of transactions, many different members of a market can meet one another's needs and so individuals can select their partners. For example, members of a market may have the same ability to provide all services, and are always available nearby, so that there is no cost to selecting a favored partner. We refer to a model of such a situation as one that allows "choice-based" matching.

[28] In particular, it can be established numerically that the sign of the derivative is always negative whenever $(\gamma_1/b(\gamma_1;\delta))b'(\gamma_1;\delta) \leq 0$ for all γ_1, where $b(\gamma_1;\delta) \equiv (\beta_{1,2} - \beta_{0,2})/(1-\delta(1-\beta_{1,2}))$.

[29] The conditional meeting probabilities of junior and senior individuals converge. N_m also affects the βs since these probabilities are also conditional on the agent surviving. These also converge toward the unconditional ones.

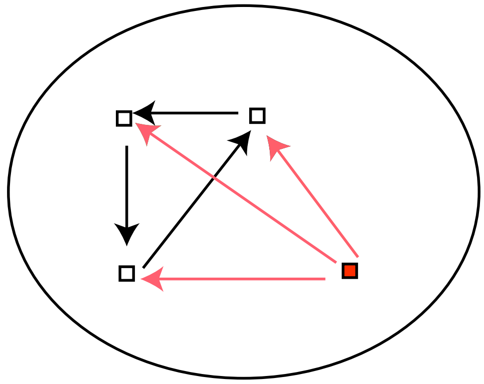


Figure 5: Simplest choice-based hierarchy: 3 seniors trust each other, and 1 junior trusts each senior with equal probability.

The choice-based model is the same as the baseline model, with the following modifications. Individuals can act as agents for as many principals as they wish. There is a public randomization device that helps assign principals to agents, to resolve agent indifferences. Although the randomization device we employ is somewhat ad hoc, we use it to simplify the analysis in the following way: we want to avoid accounting for scenarios where, as a matter of bad luck, so many principals select the same agent that the principal is tempted to "cheat and leave." Since we are endowing principals with the ability to choose their agents, it seems plausible that they would avoid overloading an agent in that way, but we simply make that part of the allocation exogenous for simplicity, and assume that principals are distributed as evenly as possible among agents while respecting agent preferences. [30]

We define a **choice-based hierarchical equilibrium** to be the same as a hierarchical equilibrium, with the following modifications. Principals select agents as follows: in period t in market j players with seniority l always select as agents the most "trustworthy" available agents, and among those, the players select those with the highest seniority.

For simplicity, we consider only candidate equilibria where γ_1 is less than 1/2. This makes sure that the number of trades that senior agents receive off the equilibrium path never exceeds the maximum number of trades they can possibly get on the equilibrium path under our assumptions about the allocation of trade.

If a choice-based hierarchical equilibrium exists with $\lambda_L = 1$ (e.g. seniors are fully trustworthy) then seniors are selected as agents and the period payoffs, for $l = 1, ..., L$ are:

$$\Pi(l) = R + 1\{l = L\}\frac{1}{\gamma_L}w,$$

[30] Formally, we assume that the public randomization device first receives request from seniors about the set of agents they consider acceptable. Then, it allocates seniors to agents, making sure that no senior is allocated to himself and that the maximal number of principals assigned to each agent is as close as feasible to the minimal number of principals assigned to any agent. Within the set of allocations that satisfies these constraints, the allocation is random. Next, the juniors specify their set of acceptable matches. The randomization device follows an analogous allocation algorithm, now considering the total number of principals assigned to each agent.

from which one can easily compute the value functions. Senior members are thus awarded a "seniority premium" which increases with the proportion of non-seniors in the hierarchy.

Consider the efficiency of such an equilibrium, if it exists (which it does for a range of parameter values). Every principal can find an agent who is trustworthy, and so trade is fully efficient. The inefficiencies due to mismatches are completely eliminated. However the value for being in the market is still increasing in seniority, due to the fact that advancements are probabilistic. As in the random matching case, the fact that value increases with seniority can deter cheat-and-leave defections.

The other potential defection is to cheat and then return to the same market the next period, knowing that they will be able to avoid the cheated principal. However, there is still a cost to the cheat-and-return defection. First, a cheater will not be selected as agent by his old partners. This reduces the expected number of jobs assigned to him in future trading periods. Second, if an individual cheats enough other principals, there is always a chance that a (big enough) *group* of previously cheated principals will fill the remaining senior positions and force the cheater to turn to more junior individuals for trust. If these sanctions are sufficiently high, then this social structure will sustain cooperation.

It is useful to note the manner in which the incentives constraints are modified from the random matching case. First, senior agents are now trusted by weakly more than one individual, and thus face a stronger period incentive to cheat them all. However, upon cheating in this way, the incentives to leave can be higher or lower relative to the hierarchical random matching model. On one hand there are more "enemies" around at home. On the other, there is more to lose by starting over: seniority is more valuable. A second issue is that the sanction if cheaters return to their home market is lower: both cheaters and cheated can choose to avoid each other and find others with whom to trade. The random matching equilibria, in contrast, relied on (correct) beliefs on the part of the cheater that the cheated individual would continue to return to his home market to deter the cheater from returning as well. This deterrent effect disappears in choice-based markets in which there are at least three seniors.

A senior is most tempted to cheat when he is assigned as an agent to the largest possible group of principals, with the lowest feasible aggregate seniority. The (IC'_{AL}) and (IC'_{AR}) constraints are thus replaced by:

$$ceil(1/(1-\gamma_L))(s-w) \leq \delta(W(\emptyset, L) - W(a, L)), \qquad (\text{IC}_A(L))$$

where "ceil(x)" denotes the largest integer greater than x, a is the set of agents with the lowest possible aggregate seniority such that $|a| = ceil(1/(1-\gamma_L))$ and $W(\{a\}, L)$ denotes the continuation payoff of a senior that has shirked on the set $\{a'\}$ in the past.[31]

Consider now a junior individual's incentives. Note that a junior, if trusted at level λ' by a

[31]As usual, we restrict to attention to profiles that map the previous period's enemy's or enemies' seniority level and the current assignment, if any (i.e. cardinality and associated seniority level of the set of individuals randomly assigned to the agent) into actions. It can be optimal to cheat only a subset of principals, to the extent that it reduces the probability that there will be no senior to trade with in subsequent periods (recall that seniors die with independent probability).

	Matched to:				
State	∅	J	S	SJ	SS
∅	*	wr	wr	wr	wr
J	*	*	wr	*	wr
S	*	wr	wr	wr	*
SJ	*	*	wr	*	*
SS	*	wr	*	*	*
SSJ	cl	*	*	*	*

(a) Seniors' equilibrium strategy

	Abs	% Institution Free
W(S)	116.667	1.16667
W(RJ)	100.502	1.00502
W(N)	99.5117	0.995117

(b) Equilibrium continuation payoffs (on the eqm path)

Period strategy	$\lambda_J = 1$	$\overline{\lambda_J}$
λ_J	1	0.970
Coop-&-R	100.492	100.462
Ch-&-R	100.451	100.391
Ch-&-L	100.522	100.462
Cheat?	Yes	no

(c) Juniors' equilibrium payoffs on the equilibrium path

	Abs	% Institution Free
V(n)	116.667	1.16667
V(J)	114.604	1.14604
V(S)	114.604	1.14604
V(SJ)	109.694	1.09694
V(SS)	109.332	1.09332
V(SSJ)	98.2098	0.982098

(d) Seniors' eqm continuation payoffs (off the equilibrium path)

Table 1: Equilibrium Profiles and Continuation Payoffs

single principal, would face the following constraint to ensure cooperation:

$$\lambda'(s-w) \leq \delta \left(W(\emptyset, 1) - W(\{l\}, 1)\right). \qquad (\text{IC}_A(1))$$

Note that the strategies we have specified entail that juniors are never selected as agents.

A symmetric equilibrium of our game is a strategy profile and a vector of continuation values satisfying the usual optimality conditions. Although it is analytically cumbersome to express conditions for existence, computationally the model is simple to solve since given a profile and a trust level for junior members, the associated continuation values solve a linear system of equations. Thus given our primitives (i.e. payoff parameters, matching function, survival and transitional probabilities) we can devise a mapping from our strategy space into itself and show conditions for existence. To demonstrate existence, we will provide a numerical example of a two level hierarchy and illustrate the crucial properties of the choice-based hierarchy.

Consider the simplest hierarchical structure that satisfies our assumptions, that is: three seniors and one junior member (See Figure 6). In this structure, there are four jobs to be randomly assigned to the seniors, i.e. a random senior is assigned two jobs. Table 1(a) shows the seniors' equilibrium profile when $w = r = R = 1$, $s = 2$ and $\delta = 0.98$. Its ij-th element denotes the action prescribed to seniors when there are i enemies around and he is assigned to subset j of principals. "*" mark contingencies which cannot occur, "wr", "cl" and "cr" stand respectively for work-and-return, cheat-and-leave, and cheat-and-return.[32] Table 1(b) reports

[32]E.g. the star in ($i = SS, j = S$) reflects the fact that a senior cannot possibly be chosen as an agent by another senior member in a (3x1) hierarchy in which two senior members do not trust him already.

the continuation payoffs on the equilibrium path of seniors, returning juniors and newcomers both in absolute terms and as a fraction of the institution free payoff $(w + r)/(1 - \delta)$. Table 1(d) reports the seniors' continuation payoffs off the equilibrium path, at the beginning of a trading period (i.e. the expected value of returning to the home market) as a function of the set of enemies present in the previous attendance spell. Lastly, Table 1(c) reports the juniors' continuation payoffs in the sub-games in which they are trusted, as a function of the action chosen and the level of trust accorded.

In this illustration, seniors are fully trustworthy while juniors are not: juniors will cheat if trusted more than $\overline{\lambda_J}$. Now consider whether this equilibrium satisfies our desired robustness properties. First, consider whether principals artificially withhold trade. There are two ways this could happen: by avoiding trustworthy agents and by not fully trusting agents to the extent that they can be trusted. In this example all trades occur at $\lambda = 1$ with seniors chosen as agents. Junior agents are not selected because they are in fact less trustworthy in equilibrium: juniors will prefer to cheat and leave whenever λ_J exceeds .97 (Table 1(c)).

Furthermore, cooperation within groups employing hierarchical strategies of this kind can be sustained even with institutional change in other groups. In the extreme case, consider a market that completely lowers its entry barriers and trusts newcomers fully and immediately, while all other markets use choice-based hierarchical equilibria. As Table (1d) shows, even in this environment, the value for seniors in the hierarchy to returning to the home market exceeds that of leaving for such an "institution-free" market. Thus, seniors always have something to lose from a cheat-and-leave strategy, no matter what their assignment is, *even if* another market trusts them immediately. Further, in our example, returning juniors also have payoffs greater than an institution-free market could provide.[33] In many environments, it is thus possible to design structures that always guarantee that their returning members receive a payoff higher than that of newcomers in an institution free group. In this way, cooperation in choice-based hierarchies can be sustained in circumstances where with other institutional arrangements, including groups based upon entry barriers, it would fail. In contrast, in an identity-investment equilibrium, new members see payoffs *lower* than they would receive in a full-trust, institution-free market, while returning members see payoffs *equal to* those in a full-trust, institution-free market. In contrast, in the choice-based hierarchical equilibrium, returning members expect payoffs *higher than* those in a full-trust, institution-free market.

Furthermore, in the example illustrated in Table 1, cooperation in the new institution-free market would also fail. If all other markets concentrate trades with seniors, an individual in an institution-free market would benefit by cheating and moving to a market following a choice-based equilibrium: $s + \delta W(N) > w + \delta(r + w)/(1 - \delta)$. Again, this is possible while still satisfying incentive constraints *within* the seniority-based markets, because part of the reward for good behavior within the seniority-based market is the prospect that returning members receive *more* than an equal share of trades.

[33] Recall that juniors are not trusted on the equilibrium path, so that we do not have to worry about cheat-and-leave defections.

We conclude the analysis by discussing how changes in the seniority structure affect existence and efficiency. Contrary to non-choice based hierarchies, there are no efficiency reasons for having bigger groups and small lower levels. Trade is always fully efficient. Thus the tradeoff between existence and efficiency ceases to exist.

On the incentives side, a greater proportion of agents in the lower levels of a hierarchy increases the return to seniority, but also decreases the probability an individual will advance to a higher level. As a result, the continuation value of non-senior individuals can increase or decrease depending on which effect predominates. Nevertheless, since seniority premia are internalized more as one rises, in the numerical examples we have studied, the "gaps" in the continuation payoffs increase with the proportion of juniors. Thus a more pyramidal hierarchy makes cheat-and-leave defections less attractive. As argued in the previous section, the temptation to cheat and leave decreases relatively more for seniors than for juniors as the proportion of juniors in the hierarchy rises. Thus designing groups with a greater proportion of agents at lower levels of the social hierarchy also reduces the trustworthiness of juniors.

6 Conclusions and Applications

This paper has analyzed the endogenous formation of groups that enable their members to sustain trust in environments where there are many possible partners, outside options are strong and legal enforcement is not available. An important set of "identity investment" institutions sustain loyalty to a group and cooperation within groups by creating barriers to starting anew. Seniority structures, however, can support loyalty and within-group cooperation without requiring artificial barriers to entry or withholding of trade. In fact, in environments where individuals can choose their trading partners, seniority structures can be designed that are robust to the entry of institution-free groups that would lead to the failure of trust in groups that are sustained by entry barriers. In fact, such seniority structures can actually undermine cooperation in institution-free groups.

The resilience of seniority- based cooperative hierarchies and the corresponding decline of trust and cooperation in institution-free groups appears to mimic a number of environments where new venues for exchange have emerged, from open source software communities to pastoral communities in West Africa. But it also hints at the dynamics of spontaneous order to expect in both historical and contemporary settings where no institutions exist. Though the formation of entry barriers based upon "cultural" distinction can sustain cooperation among sub-groups, these will be undermined by the continued presence of institution-free groups. Social hierarchies, in contrast, may survive. In the absence of third-party enforcement, hierarchies may thus emerge as a common early organizational form in new venues for trade.

A second element of the analysis worthy of note is that choice-based social hierarchies can sustain patterns of cooperative relationships that survive beyond the individuals that populate them. Thus "social networks" become less specific to the ties built up by any individual.[34] One

[34]This differs from theories of endogenous social networks (e.g. Jackson and Wolinsky [26]) in which ties are

can envisage extensions where seniors in a hierarchy effectively become marketmakers, allowing juniors to trade with one another indirectly. Our analysis hints that in new venues for exchange, characterized by large populations, weak legal systems and the ability of individuals to cheat and run, seniority-based social hierarchies may play an important role both in facilitating trust and cooperation and ultimately in creating the conditions for market development.

based upon an individual's trading history. Here centrality in the network derives from an individual's office or status as "senior". Thus, the social structure can out-live an individual agent.

Appendix

A4 Benchmarks

Proof of Proposition 4.1

Since all players perceive as equally likely to be matched will all other individuals who were alive at the end of the previous attendance spell, no matter what information set is reached and no matter what their community choices are, randomizing across markets is always optimal. Therefore we can analyze the game as if there were no markets at all.
The candidate strategies say that individuals defect with their partners *if and only if* either individual defected in past play (if any previous play). According to such strategies there are only 2 types of information sets: a) not matched with an "enemy," b) matched with an "enemy." In order to show that such strategy profile is sequentially rational we shall prove that no player has incentive to deviate once at any of these informations sets. (since $\delta < 1$ the game is continuous at infinity so we can apply the one-stage deviation principle). The condition

$$s - w \leq \frac{\delta^2}{1-\delta^2} \frac{1}{N-1} (R + w), \tag{7}$$

requires the expected discounted value of enduring cooperation with the partner to be greater or equal than the short-term gain obtained by cheating. According to the candidate profile, defecting against one individual does not affect the outcome of future interactions with third individuals, if all individuals adhere to the profile in the subgame starting after cheating. It follows that the condition is necessary and sufficient to deter cheating at information sets of type a) on and off the equilibrium path. (i.e. the condition is the same no matter how many "enemies" I had at the end of the previous attendance spell). Finally, cheating is trivially optimal at information sets of type b) as there are no current nor future gains from cooperation. Notice that the above argument is valid no matter which beliefs individuals hold about what happened in previous play between any two other individuals.

A5.1 Investments in Identity

Proof of Proposition 5.1

Suppose that any equilibrium profile satisfies the following monotonicity property: if an agent cheats if trusted in state x then he also cheats if trusted in state $x + 1$. It follows that:

Lemma 6.0.1. *Upon reaching a state x, in any equilibrium of the continuation game the agent must either (i.) cooperate and return to the home market forever thereafter or (ii.) optimize between cheating-and-returning and cheating-and-leaving in all states $x' \geq x$.*

The no shirking constraint (IC_{AR}) guarantees that the discounted value of a cooperative relationship is high enough to deter "cheat-and-return forever" defections no matter which node has been reached. Consider now the option to leave. Denoting the expected continuation payoff of the subgame starting after cheating as $W^C(1, W^L(m))$ and provided that (a) monotonicity holds, (b) $\{W^C(x, W^L(m))\}_{x=0}^{N_m-1}$ is well defined and nonincreasing in m (using the usual component-wise order), (c) $\underline{m}$ is well defined and (d) cheated principals always return (so that they are there to provide the sanction); then $m \geq \underline{m}$, is necessary and sufficient to deter type (ii.) defections and hence (IC_{AL}) and (IC_{AR}) conditions are necessary and sufficient for existence.

(b) Consider the agent's problem of whether to leave or stay after cheating in state x:

$$W^C(x, W^L) = \max_{y \in [0,1]} yW^L + (1-y) \sum_{i=0}^{N_m-1} p(i,x)[\alpha(i)[(s+R)+ \\ + \delta W^C(i+1, W^L)] + (1-\alpha(i))\delta W^C(i, W^L)], \tag{8}$$

given $W^L \geq 0$ and $W^C(0) \equiv (r+w)/(1-\delta)$. Since individuals die with independent probability, the latter is a stochastic dynamic program characterized by a convex choice set, a countable state space, a bounded payoff function, a smooth well-behaved transition function and discounting. Under these conditions, and restricting attention to equilibria in which individuals always leave whenever indifferent, an equation of (8)'s type has a unique solution (a good reference is Stokey and Lucas' (1989) ch. 9).

$[W^C(x, W^L) - W^C(x, W^{L'})]$ Since period payoffs are bounded and $\delta < 1$, equation (2) in proposition 6.1 defines a mapping, which can be denoted T, from some compact cube X living in $\mathbb{R}_+^{N_m+1}$ into itself, which is nondecreasing in W^L. Applying standard comparative statics results we conclude that the unique fixed point W^C is nondecreasing in W^L (using the usual component-wise ordering).

(c) Notice that: (i.) when $m = 0$, we should have $W^C(x, W^L) = W^L \equiv (R+w)/(1-\delta)$ for all x since $(R+w)/(1-\delta)$ is the continuation payoff on the equilibrium path. (ii.) when $m \geq (R+w)/(1-\delta)$ it should be that no individual ever leaves and hence that:

$$\frac{s-w}{\delta} \geq \frac{\delta}{1-\delta^2} \frac{1}{N_m - 1}(R+w) = W^C(x, W^L) - W^C(x+1, W^L)$$

(the latter inequality derives from the (IC_{AR}) condition being satisfied. (i.) and (ii.) together imply:

$$\frac{s-w}{\delta} \leq \frac{R+w}{1-\delta} - W^C\left(1, W^L\left(\frac{(R+w)}{1-\delta}\right)\right) = \frac{\delta}{1-\delta^2} \frac{1}{N_m - 1}(R+w)$$

$$\frac{s-w}{\delta} > \frac{R+w}{1-\delta} - W^C\left(1, W^L(0)\right) = 0.$$

By continuity and monotonicity of W^C in W^L, (IC_{AL}) has thus one and only one solution.

[**(a) and (d)**] Let $\tilde{W}(x, W^L)$ denote the continuation payoff at the *interim* stage, that is after returning and observing the number of enemies x present in the *current* attendance spell but before being matched. If an equilibrium exists then the claim is that $\tilde{W}(x, W^L)$ necessarily decreases with x. To see this, suppose the contrary. Then if an equilibrium calls for cheating in x and cooperating in $x+1$ we should have:

$$W(x+1, W^L) - W(x+2, W^L) \geq (s-w)/\delta \geq W(x, W^L) - W(x+1, W^L) \tag{9}$$

where

$$W(x, W^L) = \sum_{i=1}^{N_m - 1} p(i, x)\tilde{W}(i, W^L),$$

by definition. That (9) cannot be the case when $\delta < 1$ can be established by direct computation, a contradiction. We are left to verify that cheated principals return. Since the per period sanction to the agent for cheating is identical to the sanction to the principal when both are in the same market, if agents prefer to return to the home market when expecting principals to return, then it should be that principals also prefer to return upon being cheated.

Proof of Proposition 5.2

Since period payoffs are bounded and $\delta < 1$, (2) defines a mapping, denoted T, from some compact cube living in $\mathbb{R}_+^{N_m+1}$ into itself. $\alpha(i, N_m)$ increasing with i and decreasing with N_m imply that T is nondecreasing in N_m. It follows that its unique fixed point W^C is nondecreasing in N_m (using the standard component-wise ordering). Hence the existence region is "largest" for $N_m = 2$.

In equilibrium everybody trusts at full scale. Thus if there is no residual claimant of membership costs efficiency requires such costs (which are due every time an individual dies) to be as low as possible. A straightforward corollary of the above result is that $\underline{m}(N_m)$ is nondecreasing in N_m.

A5.2 Purgatorial Equilibrium

$W(\lambda^*)$, $W^L(\lambda^*)$ and $W^C(W^L)$

[$W(\lambda^*)$] Recall that W is computed at the interim stage and is to be interpreted as the continuation payoff of an individual who survives (i.e. who makes it to the next period) on the equilibrium path. This function should satisfy $W = E(\pi) + \delta W$, where $E(\pi)$ is the expected period payoff. Given the equilibrium profiles, $\pi = (w + R)$ if the individual is matched as a principal to a senior member and $\pi = (w + \lambda^* R)$ if not. When $t^* = 1$ the expected number of individuals in purgatory in a community is equal to the expected number of newcomers. Since individuals die with independent probability $(1 - \delta)$, conditional on surviving, a returning member expects a fraction $(1 - \delta)$ of the residual $(N_m - 1)$ members to be replaced. Hence

$W(\lambda^*)$ solves:

$$(1-\delta)W(\lambda^*) = (w + \delta R + (1-\delta)\lambda^* R)\,. \tag{10}$$

$[W^L(\lambda^*)]$ Newcomers replace old members that have either died or left. Thus, on the equilibrium path, the expected number of openings in a generic community will be equal to the expected number of deaths $(1-\delta)N_m$. Then the subjective probability of being matched with another newcomer is:

$$\frac{(1-\delta)N_m}{N_m - 1} - \frac{1}{N_m - 1} + \varepsilon(N_m),$$

which is the expected number of newcomers (other than oneself) over the number of potential matches. The correction $\varepsilon(N_m)$ is necessary since the mere fact that "there is one opening" in a community is informative about the total number of openings.[35] Letting $p(i, N)$ denote the probability that i out of N die, we can denote the expected number of deaths by $D \equiv \sum_{i=1}^{N_m} \tilde{p}(i, N_m)i$ where $\tilde{p}(i, N_m) = p(i, N_m)/(1 - p(0, N_m))$. It follows that $D = (1-\delta)N_m/(1-\delta^{N_m})$ and therefore the probability of being matched with a newcomer upon joining a community is $(D-1)/(N_m - 1)$.

The value for an individual beginning purgatory is thus:

$$W^L(\lambda^*) \equiv \frac{(1-\delta)N_m - 1 + \delta^{N_m}}{(1-\delta^{N_m})(N_m - 1)}(\lambda^* w + \lambda^* R) + \frac{N_m(\delta - \delta^{N_m})}{(N_m - 1)(1-\delta^{N_m})}(\lambda^* w + R) + \delta W(\lambda^*)$$

Proof of proposition 5.3

If condition (3) is not satisfied, then by definition of $\underline{m}$ an individual can always gain by cheating and leaving the home market. The following proposition provides a more general necessary and sufficient condition:

Proposition 5.3b *Suppose that for some δ, N, r, R and s an Identity Investment equilibrium exists with J active markets, then a Purgatorial equilibrium exists with the same number of markets if and only if there is a λ^* in $[0, 1]$ such that*

$$(s - w) \leq \delta(W(\lambda^*) - W^C(1, W^L(\lambda^*))). \tag{IC_{AL}}$$

where, as in the previous section, $W^C(1, W^L)$ denotes the seniors' continuation payoff in the subgame starting after cheating which is solution to an analogous system of linear equations (one only needs to substitute in the right period payoffs).

Finally notice that since leaving is always an option in the continuation game starting after cheating, $W^C(1, W^L(\lambda^*))$ is trivially bigger than $W^L(\lambda^*)$ and hence the above condition is weakly stronger than the one provided in the paper.

[35] In guessing the number of heads that came out of a toss of N fair coins ($i.e. N/2$), Bayesian updaters will revise upward their estimate when told that the event "all tosses are tails" did not occur. (e.g. if $N = 1$ then the estimate goes up from .5 heads to 1.)

A5.3 Hierarchical Equilibrium

Preliminaries: Transition probabilities

Assuming a 2-tier hierarchy, we sum over possible combinations of dead level 1 and 2 agents to get the full expression for $\beta_{1,2}$, taking into account that one of the level 1 cannot die. k represents dead seniors, and l represents dead juniors. Given k and l, the probability of advancement for a given junior conditional on his survival is the number of available slots, k, divided by the number of surviving juniors who could be promoted, $\gamma_1 N_m - l$.

$$\beta_{1,2} = \sum_{k=0}^{\gamma_2 N_m} \sum_{l=0}^{\gamma_1 N_m - 1} \left[(1-\delta)^{k+l}\,\delta^{N_m-1-k-l}\right] \cdot \begin{pmatrix} \gamma_1 N_m - 1 \\ l \end{pmatrix} \cdot \begin{pmatrix} \gamma_2 N_m \\ k \end{pmatrix} \cdot \min\left\{1, \frac{k}{\gamma_1 N_m - l}\right\}.$$

Similarly we sum over possible combinations of dead level 1 and 2 agents to get the full expression for $\tilde{\beta}_{1,2}(L)$, taking into account that one agent of level 1 and one agent of level L cannot die (i.e. the probability is conditional on them surviving).

$$\tilde{\beta}_{1,2}(2) = \sum_{k=0}^{\gamma_2 N_m - 1} \sum_{l=0}^{\gamma_1 N_m - 1} \left[(1-\delta)^{k+l}\,\delta^{N_m-2-k-l}\right] \cdot \begin{pmatrix} \gamma_1 N_m - 1 \\ l \end{pmatrix} \cdot \begin{pmatrix} \gamma_2 N_m - 1 \\ k \end{pmatrix} \cdot \min\left\{1, \frac{k}{\gamma_1 N_m - l}\right\}$$

Lastly we do the same to get the expression for $\beta_{0,2}$, the only difference being that one of the sum starts from one to avoid the expression from being indefinite for $k = l = 0$

$$\beta_{0,2} = \sum_{k=1}^{\gamma_2 N_m} \sum_{l=0}^{\gamma_1 N_m} \left[(1-\delta)^{k+l}\,\delta^{N_m-k-l}\right] \cdot \begin{pmatrix} \gamma_1 N_m \\ l \end{pmatrix} \cdot \begin{pmatrix} \gamma_2 N_m \\ k \end{pmatrix} \cdot \max\left\{0, \frac{k+l-\gamma_1 N_m}{k+l}\right\}.$$

Proof of proposition 5.6

We shall sign:

$$\frac{dW/2N_m}{d\gamma_1} = -(1-\lambda_1^*)(R+w) + \frac{d\lambda_1^*}{d\gamma_1}\gamma_1(R+w). \tag{11}$$

λ_1^* solves the juniors' (IC_{AL}) with equality and is thus equal to

$$\lambda_1^* = \frac{w - \frac{R}{N_m-1}b}{\frac{s-w}{\delta} + (w - \frac{R}{N_m-1})b}, \tag{12}$$

where b is $b(\gamma_1;\delta) \equiv (\beta_{1,2} - \beta_{0,2})/(1-\delta(1-\beta_{1,2}))$. Assuming that the partial derivative exists we get:

$$\frac{d\lambda^*}{d\gamma_1} = \frac{\frac{db}{d\gamma_1}(w - \frac{R}{N_m-1})(\frac{s-w}{\delta})}{((w - \frac{R}{N_m-1})b + (\frac{s-w}{\delta}))^2}. \tag{13}$$

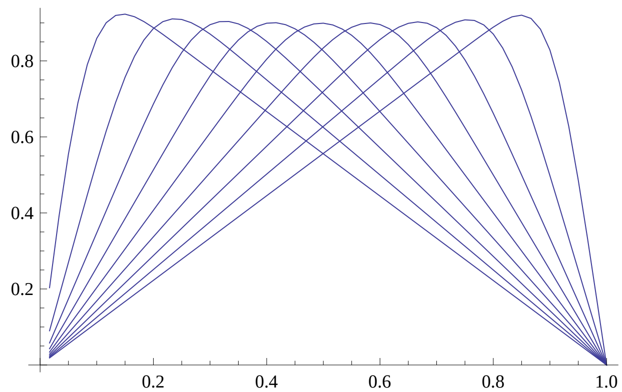


Figure 6: Plot of $b(\gamma_1, \delta)$ for values of δ ranging from 1/10 to 9/10 with 1/10 increments.

It follows that

$$\text{sign}\left(\frac{dw}{d\gamma_1}\right) = \text{sign}\left(-(1-\lambda_1^*) + \frac{\frac{db}{d\gamma_1}(w - \frac{R}{N_m-1})(\frac{s-w}{\delta})}{((w-\frac{R}{N_m-1})b + (\frac{s-w}{\delta}))^2}\gamma_1\right)$$

$$= \text{sign}\left(\left(w - \frac{R}{N_m - 1}\right)\left(\frac{db}{d\gamma_1}\gamma_1 - b\right) - \frac{s-w}{\delta}\right).$$

Since the latter term is always positive, a sufficient condition for the sign to be negative is:

$$\frac{\frac{db}{d\gamma_1}\gamma_1}{b} \leq 0 \text{ for all } \gamma_1 \in [0,1] \tag{14}$$

We have a numerical proof of the above result that exploits the fact that the condition depends only on δ and γ_1 which are both bounded in $[0,1]$and can be easily verified numerically on a grid. Since the condition can be shown to be formally equivalent to the b function being strictly concave in γ_1 we can intuitively show our numerical results with a simple plot (figure 1).

References

[1] George A. Akerlof and Rachel E. Kranton. Economics and identity. *Quarterly Journal of Economics*, 115(3):715–53, 2000.

[2] George A. Akerlof and Rachel E. Kranton. Identity and schooling: some lessons for the economics of education. *Journal of Economic Literature*, 40:1167–1201, December 2002.

[3] Alberto Alesina and Eliana La Ferrara. Ethnic diversity and economic performance. *Journal of Economic Literature*, 43(3):762–800, 2005.

[4] Abhijit Banerjee and Esther Duflo. Reputation effects and the limits of contracting: a study of the Indian software industry. *Quarterly Journal of Economics*, 115(3):989–1017, August 2000.

[5] Robert H. Bates. *Essays on the political economy of rural Africa*, chapter 2, pages 21–59. Cambridge University Press, Cambridge, 1983.

[6] Gary S. Becker and George J. Stigler. De gustibus non est disputandum. *American Economic Review*, 67(2):76–90, March 1977.

[7] Alberto Bisin and Thierry Verdier. "Beyond the melting pot": cultural transmission, marriage, and the evolution of ethnic and religious traits. *Quarterly Journal of Economics*, 115(3):955–988, August 2000.

[8] Simon Board. Relational contracts and the value of loyalty. mimeo, UCLA, February 2008.

[9] H. L. Carmichael and W.B. Macleod. Gift giving and the evolution of cooperation. *International Economic Review*, 38:485–508, 1997.

[10] Avinash K. Dixit. Trade expansion and contract enforcement. *Journal of Political Economy*, 111(6):1293–1317, December 2003.

[11] Jean Ensminger. *Making a market*. Cambridge University Press, Cambridge, UK, 1992.

[12] Joan Esteban and Debraj Ray. On the salience of ethnic conflict. *American Economic Review*, 98(5):2185–2202, 2008.

[13] Marcel Fafchamps. The enforcement of commercial contracts in Ghana. *World Development*, 24(3):427–448, 1996.

[14] Eric Friedman and Paul Resnick. The social cost of cheap pseudonyms. *Journal of Economics and Management Strategy*, 10(2):173–199, 2001.

[15] Roland G. Fryer Jr. An economic approach to cultural capital. mimeo, University of Chicago, October 2002.

[16] Parikshit Ghosh and Debraj Ray. Cooperation in community interaction without information flows. *Review of Economic Studies*, 63(3):491–519, July 1996.

[17] Robert Gibbons. What is economic sociology and should any economists care? *Journal of economic perspectives*, 19(1):3–7, Winter 2005.

[18] Robert Gibbons, Richard Holden, and Michael Powell. Integration and information: markets and hierarchies revisited. mimeo, MIT, January 2010.

[19] Joseph Greenberg. Islam and clan organization among the Hausa. *Southwestern Journal of Anthropology*, 3(3):193–211, Autumn 1947.

[20] Avner Greif. Contract enforceability and economic institutions in early trade: the Maghribi traders' coalition. *American Economic Review*, 83(3):525–547, June 1993.

[21] Michael T. Hannan, Laszlo Polos, and Glenn R. Carroll. *Logics of Organization Theory: Audiences, Code, and Ecologies.* Princeton University Press, Princeton, 2007.

[22] Oliver Hart and John Moore. On the design of hierarchies: coordination versus specialization. *Journal of Political Economy*, 113:675–102, 2005.

[23] Benjamin Hermalin. Leadership and corporate culture. In Robert Gibbons and John Roberts, editors, *Handbook of Organizational Economics*. Princeton University Press, 2010.

[24] Robin Horton. Stateless societies in the history of West Africa. In J.F.A. Ajayji and Michael Crowder, editors, *The History of West Africa*, volume 1, chapter 3, pages 72–114. Columbia University Press, New York, 1976.

[25] Laurence R. Iannaccone. Sacrifice and stigma: reducing free-riding in cults, communes and other collectives. *Journal of Political Economy*, 100(2):271–291, April 1992.

[26] Matthew O. Jackson and A. Wolinsky. A strategic model of social and economic networks. *Journal of Economic Theory*, 71:44–74, 1996.

[27] Saumitra Jha. Trade, institutions and religious tolerance: evidence from india. GSB Research Paper 2004, Stanford Graduate School of Business, Stanford CA, January 2008.

[28] Michihiro Kandori. Social norms and community enforcement. *Review of Economic Studies*, 59:63–80, 1992.

[29] Benjamin Klein and Kenneth B. Leffler. The role for market forces in assuring contractual performance. *Journal of Political Economy*, 89(4):615–41, 1981.

[30] Rachel E. Kranton. The formation of cooperative relationships. *Journal of Economics, Law and Organisation*, 12(1):214–33, April 1996.

[31] David M. Kreps. Corporate culture and economic theory. In James E. Alt and Kenneth A. Shepsle, editors, *Perspectives on Positive Political Economy*, pages 90–143. Cambirdge University Press, Cambridge, 1990.

[32] Murray Last. Some economic aspects of conversion in Hausaland. In Nehemia Levitzion, editor, *Conversion to Islam*, chapter 13, pages 236–246. Holmes and Meier, New York, 1979.

[33] Joshua Lerner and Jean Tirole. The economics of technology sharing: open source and beyond. *Journal of Economic Perspectives*, 19(2):99–120, Spring 2005.

[34] John Lindsey, Benjamin Polak, and Richard Zeckhauser. Free love, fragile fidelity and forgiveness: rival social conventions under hidden information. mimeo, October 2003.

[35] Dalia Marin and Monika Schnitzer. Disorganization and financial collapse. In Erik Bergloef and Gerard Roland, editors, *The economics of transition: the fifth Nobel Symposium in economics*, pages 70–94. Palgrave Macmillan, 2007.

[36] Eric Maskin, Yingyi Qian, and Chenggang Xu. Incentives, information and organizational form. *Review of Economic Studies*, 67:359–78, 2000.

[37] John McMillan. *Reinventing the Bazaar: a natural history of markets.* W.W. Norton & Co., New York, 2002.

[38] John McMillan and Christopher Woodruff. The central role of entrepreneurs in transition economies. *Quarterly Journal of Economics*, 114:1285–1320, November 1999.

[39] John McMillan and Christopher Woodruff. Interfirm relationships and informal credit in Vietnam. *Quarterly Journal of Economics*, 114:1285–1320, November 1999.

[40] Martin A. Nowak and Karl Sigmund. Evolution of indirect reciprocity. *Nature*, 437:1291–1298, October 2005.

[41] Daniel Posner. The political salience of cultural difference: Why chewas and tumbukas are allies in zambia and adversaries in malawi" american political science review, 98(4)(2004) the political salience of cultural difference: Why chewas and tumbukas are allies in zambia and adversaries in malawi. *American Political Science Review*, 98(4):529–545, 2004.

[42] Garey Ramey and Joel Watson. Bilateral trade and opportunism in a matching market. *Contributions to theoretical economics*, 1(1), 2001.

[43] Debraj Ray. Aspirations, poverty and economic change. unpublished manuscript, March 2003.

[44] Eric S. Raymond. *The Cathedral and Bazaar: musings on linux and open source by an accidental revolutionary.* O' Reilly, Cambridge, MA, 2001.

[45] Sonali Shah. Motivation, governance and the viability of hybrid forms in open source software development. *Management Science*, 52(7):1000–1014, 2006.

[46] Joel Sobel. A theory of credibility. *Review of Economic Studies*, 52(4):557–73, October 1985.

[47] Joel Sobel. For better or forever: formal versus informal enforcement. *Journal of Labor Economics*, 2006.

[48] Eric Van den Steen. On the origin of shared beliefs (and corporate culture). MIT Sloan working paper 4553-05, August 2005.

[49] Timothy Van Zandt. Real-time decentralized information processing as a model of organizations with boundedly rational agents. *Review of Economic Studies*, 66:633–58, 1998.

[50] Joel Watson. Starting small and renegotiation. *Journal of Economic Theory*, 85:52–90, 1999.

[51] Harrison C. White. *Chains of opportunity: system models of mobility in organizations.* Harvard University Press, Cambridge, MA, 1970.

[52] Christopher Woodruff. Contract enforcement and trade liberalization in Mexico's footwear industry. *World Development*, 26(6):979–991, 1998.